7. Vectors

- Let $\vec{a_1}, \vec{a_2}, \vec{a_3}, ..., \vec{a_n}$ be n vectors. Let the linear combination of these vectors be denoted by $\vec{L}$. Then:

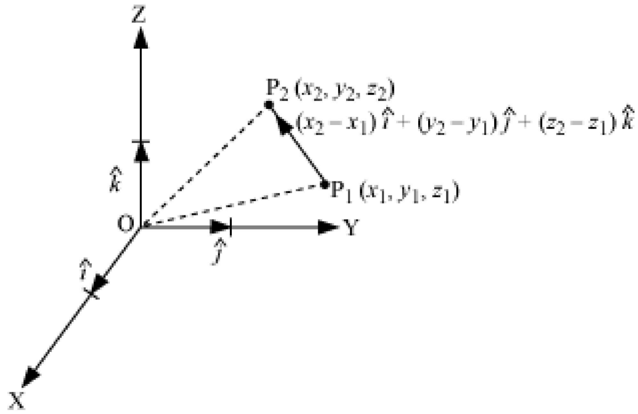
$\vec{L} = x_1 \vec{a_1} + x_2 \vec{a_2} + x_3 \vec{a_3}, ... + x_n \vec{a_n}$, where $x_1, x_2, x_3, ..., x_n \in R$

- If $x_1 \vec{a_1} + x_2 \vec{a_2} + x_3 \vec{a_3}, ... + x_n \vec{a_n} = 0$ such that not all $x_1, x_2, x_3, ..., x_n \in R$ are zero, then it can be said that $\vec{a_1}, \vec{a_2}, \vec{a_3}, ..., \vec{a_n}$ are linearly dependent vectors.
- If $x_1 \vec{a_1} + x_2 \vec{a_2} + x_3 \vec{a_3}, ... + x_n \vec{a_n} = 0 \Rightarrow \vec{a_1} = \vec{a_2} = \vec{a_3} ... \vec{a_n} = 0$, then $\vec{a_1}, \vec{a_2}, \vec{a_3}, ..., \vec{a_n}$ are linearly independent vectors.
- Let $\vec{a}, \vec{b}$ be two vectors and there exist a scalar $x \in R$ such that $\vec{a} = x \vec{b}$. Then we can say that the two vectors $\vec{a}, \vec{b}$ are collinear.
- Let $\vec{a_1}, \vec{a_2}, \vec{a_3}$ be three vectors and there exist three scalars $x_1, x_2, x_3 \in R$, not all zero such that $x_1 \vec{a_1} + x_2 \vec{a_2} + x_3 \vec{a_3} = 0$, where $x_1 + x_2 + x_3 = 0$. Then we can say that the three vectors $\vec{a_1}, \vec{a_2}, \vec{a_3}$ are collinear.
- Let *A, B, C* be three collinear points. Then each pair of the vectors $\vec{AB}, \vec{BC}$; $\vec{AB}, \vec{AC}$; and $\vec{BC}, \vec{AC}$ is a pair of collinear vectors. Thus, to check the collinearity of three points, we can check the collinearity of any two vectors obtained with the help of three points.
- Three points with position vectors $\vec{a}, \vec{b}, \vec{c}$ are collinear, only if there exist three scalars x, y, z, not all zero simultaneously such that $x\vec{a} + y\vec{b} + z\vec{c} = \vec{0}$, together with $x + y + z = 0$.
- Let $\vec{a_1}, \vec{a_2}, \vec{a_3}, \vec{a_4}$ be three vectors and there exist three scalars $x_1, x_2, x_3, x_4 \in R$, not all zero such that $x_1 \vec{a_1} + x_2 \vec{a_2} + x_3 \vec{a_3} + x_4 \vec{a_4} = 0$, where $x_1 + x_2 + x_3 + x_4 = 0$. Then we say that the three vectors $\vec{a_1}, \vec{a_2}, \vec{a_3}, \vec{a_4}$ are coplanar.

Vector Joining Two Points

The vector joining two points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$, represented as $\overrightarrow{P_1P_2}$, is calculated as

$$\overrightarrow{P_1P_2} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$



The magnitude of $\overrightarrow{P_1P_2}$ is given by $\left|\overrightarrow{P_1P_2}\right| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Section Formula

If point *R* (position vector $\vec{r}$) lies on the vector $\overrightarrow{PQ}$ joining two points *P* (position vector $\vec{a}$) and *Q* (position vector $\vec{b}$) such that *R* divides $\overrightarrow{PQ}$ in the ratio *m: n* $\left[\text{i.e.} \frac{\overrightarrow{PR}}{\overrightarrow{RQ}} = \frac{m}{n}\right]$

Internally, then $\vec{r} = \dfrac{m\vec{b} + n\vec{a}}{m + n}$

Externally, then $\vec{r} = \dfrac{m\vec{b} - n\vec{a}}{m - n}$

- **Scalar Triple Product**

The scalar triple product of the three vectors a⟶, b⟶, c⟶, defined by a⟶· b⟶× c⟶ is a scalar quantity.

a⟶·b⟶× c⟶=a1a2a3b1b2b3c1c2c3

The scalar triple product, a⟶· b⟶× c⟶ can be denoted by a⟶ b⟶ c⟶**.**

Remarks :

1. a⟶ b⟶ c⟶ =b⟶ c⟶ a⟶ =c⟶ a⟶ b
2. a⟶ b⟶ c⟶ =-b⟶ a⟶ c⟶ =-a⟶ c⟶ b⟶
3. a⟶+b⟶ c⟶ d⟶=a⟶ c⟶ d⟶+b⟶ c⟶ d⟶
4. a⟶ b⟶ c⟶=0 if a⟶=b⟶ or b⟶=c⟶ or c⟶=a⟶ or atleast one of the vector is a null vector.

5. Three vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar if and only if $\vec{a}\ \ \vec{b}\ \ \vec{c} = 0$.

6. $l\vec{a}\ \ m\vec{b}\ \ n\vec{c} = lmn\vec{a}\ \ \vec{b}\ \ \vec{c}$, where l, m and n are scalars.

- Volume of the parallelopiped whose concurrent edges are $\vec{a}, \vec{b}, \vec{c}$ is $\vec{a} \cdot \vec{b} \times \vec{c} = \vec{a}\ \ \vec{b}\ \ \vec{c}$.

- If $\vec{a}, \vec{b}, \vec{c}$ represents three adjacent edges of a tetrahedron, then its volume V is given by $V = 16\vec{a}\ \ \vec{b}\ \ \vec{c}$.

- The vector product of $\vec{a}$ with $\vec{b} \times \vec{c}$ is the vector triple product of the vectors $\vec{a}, \vec{b}, \vec{c}$ and is defined by $\vec{a} \times \vec{b} \times \vec{c}$. This is vector in the plane of $\vec{b}$ and $\vec{c}$ and perpendicular to $\vec{a}$.

  $\vec{a} \times \vec{b} \times \vec{c} = \vec{a} \cdot \vec{c}\vec{b} - \vec{a} \cdot \vec{b}\vec{c}$.

- If the vectors $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular i.e., $\vec{a} \cdot \vec{c} = 0$, $\vec{a} \cdot \vec{b} = 0$, $\vec{b} \cdot \vec{c} = 0$, then $\vec{a} \times \vec{b} \times \vec{c} = 0$.

- If the vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar then $\vec{a} \times \vec{b} \times \vec{c} = 0$.

- Let $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ be four vectors then scalar product of these vectors is defined as $\vec{a} \times \vec{b} \cdot \vec{c} \times \vec{d}$.

$$\vec{a} \times \vec{b} \cdot \vec{c} \times \vec{d} = \vec{a} \cdot \vec{c}\ \vec{b} \cdot \vec{d} - \vec{a} \cdot \vec{d} \cdot \vec{b} \cdot \vec{c} = \vec{a} \cdot \vec{c}\ \ \vec{a} \cdot \vec{d}\vec{b} \cdot \vec{c}\ \ \vec{b} \cdot \vec{d}$$

- Let $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ be four vectors then vector product of these vectors is defined as $\vec{a} \times \vec{b} \times \vec{c} \times \vec{d}$.

$$\vec{a} \times \vec{b} \times \vec{c} \times \vec{d} = \vec{a} \times \vec{b} \cdot \vec{d}\vec{c} - \vec{a} \times \vec{b} \cdot \vec{c}\vec{d}$$
$$\Rightarrow \vec{a} \times \vec{b} \times \vec{c} \times \vec{d} = \vec{a}\ \ \vec{b}\ \ \vec{d}\vec{c} - \vec{a}\ \ \vec{b}\ \ \vec{c}\vec{d}$$